CERTIFICATE OF APPROVAL

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Draven W. Houser

ENTITLED

A New Quantum Eraser Using Hyperentangled Photons

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR DEPARTMENTAL HONORS IN PHYSICS

Hyung S. Choi

Chair: Hyung S. Choi, Ph.D.

Committee on Thesis and Final Examination

Dongxue Zhao, Ph.D.

Darrel Iler, Ph.D.

A New Quantum Eraser Using Hyperentangled Photons

****

A Thesis
Presented to the Faculty
of
Greenville University

****

Submitted in partial fulfillment
of the requirements
for
Departmental Honors in Physics
by
Draven W. Houser
2019

Contents

Acknowledgements

Firstly, I wish to thank my dear friend and research partner Dustin Swarm, whose tremendous contribution to both the undergraduate research behind the topic of this thesis and the thesis itself cannot be understated.

I also wish to thank Dr. Hyung Choi, my mentor, advisor, and teacher, who has been a continual inspiration for me in class, in research, and in living as a Christian scientist. His patience with and encouragement of my numerous inquiries in physics and other areas have had a drastic impact upon me academically, personally, and professionally.

Finally, I wish to thank Dr. Zhao and Dr. Iler both for their instruction and for serving on my thesis committee.

Abstract

A new type of quantum eraser using hyperentangled photon pairs is proposed. Whereas a typical quantum eraser makes use of photons that are entangled in a single quantum state, usually in polarization, our new quantum eraser exploits entanglements in both polarization and momentum states. In a typical quantum eraser one gains "which-path" information of one photon by changing the polarization measurement of the other. This results in "erasing" the interference pattern previously obtained through indistinguishability of paths. In our new quantum eraser we gain information about the momentum state of one photon by changing the polarization of the other photon in one of its momentum states. The knowledge of the momentum state then "erases" the interference pattern previously obtained in the coincident counts of the photon pairs. This new quantum eraser may readily be implemented using photon pairs produced by Type-I Parametric Down Conversion. To achieve extra entanglement in momentum states, we simply subject them to pass through two distinguishable sets of pinholes and then recombine them before they reach polarization detectors.

Chapter 1

Introduction

Premise

A quantum eraser is a device that allows observation of how measurement of one particle in a two-particle system can affect measurement of the other particle. By changing what is known about one particle, denoted as the signal, the interference pattern measured by changing the phase of the other particle, denoted as the idler, can be "erased" or brought back. In this experiment, photons entangled in polarization and momentum (hyperentangled photons) are used to affect this interference pattern based upon what is known about the photons. By using hyperentanglement, it is possible to erase the interference pattern obtained via polarization measurement by gaining knowledge about the momentum from the signal photon's polarization measurement. This is significant in that rather than entanglement in one degree, which is generally used in quantum erasers, this uses hyperentanglement, and is thus a new type of quantum eraser.

Entanglement

Entanglement is a phenomenon that occurs when two particles, such as photons, interact in a way that their quantum states cannot be described independently of each other. In experiments, this can be ensured by giving them the same initial conditions. The result of entanglement is that any measurement on one particle will have an immediate effect on the other such that the quantum state of both particles can be determined via the measurement of only one; i.e. measuring one particle collapses the wave state of both the measured and unmeasured particles. Quantum entanglement leads to the possibility of destroying the interference pattern that might be obtained from a normal double slit or quantum eraser experiment by providing information on one particle based on information gained from another.

As an example, if two photons are entangled in terms of their polarization, knowing the polarization of one photon allows us to know the polarization of the other. As a result, the wave function of the unmeasured photon collapses. It now has a different expectation value for its polarization measurement without having been tampered with itself.

Parametric Down-Conversion

In spontaneous parametric down-conversion (SPDC) a nonlinear crystal splits a single photon into two photons of approximately equal energy, the sum of which is equal to the energy of the original photon. The daughter photons also have the same phase and closely correlated polarizations. There are two types of SPDC: Type I produces daughter photons with identical linear polarizations (described by the wave function $|\psi\rangle = \frac{1}{\sqrt{2}}\{|H,H\rangle + |V,V\rangle\}$), and Type II produces daughter photons with orthogonal linear polarizations (shown by the wave function $|\psi\rangle = \frac{1}{\sqrt{2}}\{|H,V\rangle + |V,H\rangle\}$). The original photon is called the "pump" photon and the resulting pair is called the "signal" and "idler" photons. Because the down-conversion process is a result of random vacuum fluctuations within the crystal, the production of signal and idler photon pairs is completely random. In our experiment, SPDC is achieved by sending a pump laser beam into a beta-barium borate (BBO) crystal, which results in Type I SPDC with the signal and idler photons moving in an approximately 3 degree cone extending from the crystal. Each pair of signal and idler photons is diametrically opposed on a cross sectional area of the cone due to momentum conservation.

Polarization and Quantum Erasers

The effects of photon entanglement have given rise to a unique type of experiment known as the quantum eraser. Quantum erasers demonstrate the role measurement plays in the physical reality observed.

In one form of traditional quantum eraser experiment, a pair of entangled photons are produced through SPDC and sent in differing directions. The signal photon (γ_s) is sent straight to a detector (D_s), while the idler photon (γ_i) is directed toward a Mach-Zehnder interferometer using linear polarizing beam splitters (PBS) to split and recombine the photon pathway.

Polarizing beam splitters consist of two pieces of birefringent material fused together to create a cube. Between the fused faces is a partially reflective coating that allows photons that are horizontally polarized to be transmitted and photons that are vertically polarized to be reflected. Thus, if γ_i is horizontally polarized it will travel one arm of the interferometer, whereas it will travel the other arm of the interferometer if it is vertically polarized. The horizontal polarization path and the vertical polarization path are recombined using another PBS and sent to a detector (D_i). This recombination makes it impossible to determine which path γ_i traveled.

Adjusting the mirrors along each arm of the interferometer varies the path-length for each arm, creating phase differences in the wave function of the interferometer. Due to the uncertainty of which path γ_i travels through the Mach-Zehnder interferometer, it can be considered to have travelled through both paths simultaneously, allowing for self-interference. This interference pattern is apparent in the coincidence counts between D_s and D_i . For some phase differences, a photon arriving at D_s will be accompanied by its entangled partner arriving at D_i , while for other phase differences a photon arriving at D_s will have no coincident measurement of a photon at D_i. The expectation value for γ_i is $\langle\gamma_i\rangle = \cos\phi$, where ϕ is the phase difference of the Mach-Zehnder interferometer (see section 3.2 for a comparable calculation of the expectation value).

Consider now an experimental design altered so that D_s was not just measuring the arrival of a photon but also the linear (horizontal/vertical) polarization of γ_s. Because the photon pair was created through SPDC, measuring the polarization of γ_s would give the observer information about the polarization of γ_i , even if the polarization of γ_i were not directly measured. For instance, in Type I SPDC, a measurement on γ_s showing it were horizontally polarized would tell us that γ_i was also horizontally polarized. Being able to determine the polarization of γ_i would additionally indicate which path the photon took through the interferometer. Gaining this "which-path" information through the polarization measurement of γ_s results in a loss of the interference pattern observed at D_i. Varying the phase in the Mach-Zehnder interferometer no longer has an effect on the coincidence measurements, and we find $\langle\gamma_i\rangle = 0$. Even though no polarization measurement is made on γ_i, the knowledge of γ_s's polarization results in γ_i adopting particle-like behavior and traveling only one arm of the interferometer at a time.

Now, instead of measuring the linear polarization of γ_s we measure the circular polarization of γ_s. The ket vector representation for a right-handed polarized photon is $|R\rangle = \frac{1}{\sqrt{2}}\{|H\rangle + i\,|V\rangle\}$, while the representation for a left-handed polarized photon

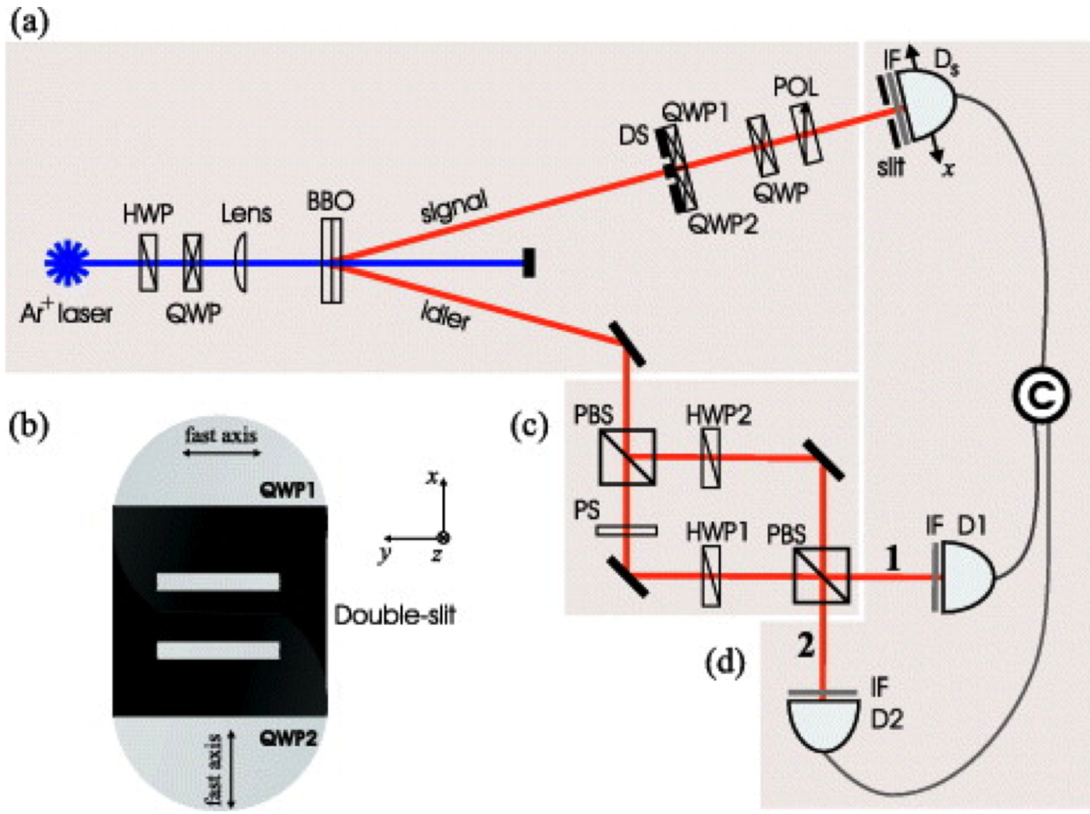

Figure 1.1: Above is shown a diagram of a quantum eraser utlizing a Mach-Zehnder interferometer. In this scheme, the photon source is an argon laser. The beam travels through a half and quarter wave plate and a lens before reaching the BBO crystal. At the crystal, SPDC occurs to create the signal and idler entangled photons. The signal photon heads directly to a detector through two quarter wave plates and a PBS, so the polarizaton can be affected as needed for this photon. The idler is directed to the Mach-Zehnder interferometer, where the idler photon's self-interference is measured by detectors D1 and D2. However, once the quarter wave plates pertaining to the signal photon's path are adjusted so that its polarization corresponds with that of the idler, the idler photon's interference pattern disappears.

is $|L\rangle = \frac{1}{\sqrt{2}}\{|H\rangle - i\,|V\rangle\}$. Solving for $|H\rangle$ and $|V\rangle$ in terms of $|R\rangle$ and $|L\rangle$ yields

$$|H\rangle = \frac{1}{\sqrt{2}}\{|R\rangle + |L\rangle\} \tag{1.1}$$

$$|V\rangle = \frac{-i}{\sqrt{2}}\{|R\rangle - |L\rangle\}. \tag{1.2}$$

Substituting these vectors, Type I SPDC the wave function of the entangled photon pair is given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|RL\rangle + |LR\rangle\}. \tag{1.3}$$

As shown by equation (1.3), measuring the circular polarization of γ_s will still give the polarization of γ_i (e.g. γ_s is measured to be right-handed polarized, indicating that γ_i is left-handed polarized). However, knowing the circular polarization of γ_i does not give any indication for which path γ_i took through the interferometer because the circular polarizations are a mixture of both horizontal and vertical polarization. This erases the "which-path" information and reintroduces the interference pattern at D$_i$.

Chapter 2

Quantum Eraser with Hyperentanglement

Hyperentanglement

As previously explained in section 1.1, photons (and other particles) are considered to be entangled when the wave functions of each photon cannot be factored or simplified. Their wave functions are connected. It is most common to see entanglement based on polarization, as seen in the entangled photon pairs produced by SPDC (Type I or Type II).

However, photons need not have only one degree of entanglement. In many experimental setups using entangled photon pairs produced through SPDC, detectors are placed to collect a photon pair that travels along one specific path. Consider instead an experimental design that selected photon pairs from two different paths and then recombined them in such a way that the detectors cannot determine which path the photons travelled.

Such an experimental design would require the wave equations to incorporate not only entanglement in polarization, but also entanglement in momentum. Entanglement in two or more degrees is known as *hyperentanglement*. Hyperentanglement allows for more robust tests of the properties of entangled photons and interesting new experimental designs.

Experimental Design

In our quantum eraser we employ a paired BBO crystal to induce Type I SPDC of a 405 nm pump photon into 810 nm signal and idler photons. As previously stated, these photons travel along a path 3 degrees on either side of the normal from the surface of the crystal in which they were produced. This means that the photon pair could travel anywhere in a circular cone emanating from the BBO.

In order to achieve hyperentanglement, a screen with four pinholes is placed in the path of the cone, selecting two specific paths for the photon pairs to travel (denoted here as path k and path l). Paths k and l are then reflected back along converging trajectories by an array of four mirrors mounted on translational stages. While on these converging trajectories, the idler photon's paths pass through two half-wave plates (HWP_k and HWP_l).

The converging paths are directed to a biconcave lens such that they emerge from the lens on parallel trajectories. These trajectories are close enough to one another that the two paths are indistinguishable to a detector, producing entanglement in momentum. After exiting the lenses, the pump and idler pass through a linear PBS that directs the photons to detectors.

The lenses used in this experiment are biconcave lenses with an effective focal length of -0.625 cm. The mirror array is located 109.22 cm (43 in) down the table from the face of the BBO crystal. The mirrors are angled such that the photons will travel back along a path of identical distance as that from the BBO to the array. That is, the surface of each mirror is angled such that each photon has a 3° angle of incidence and is reflected back along a plane that is orthogonal to the surface of the mirror.

This reflection results in path k and path l on the signal and idler sides of the pump beam converging at a point on the same horizontal and vertical plane as the BBO crystal, but 10.16 cm (4 in) on either side of the BBO crystal. Because the biconcave lenses have a focal length of -0.625 cm, placing the lenses 0.625 cm downtable from this new convergence point results in the photon paths emerging from the lenses on trajectories parallel to the surface of the table.

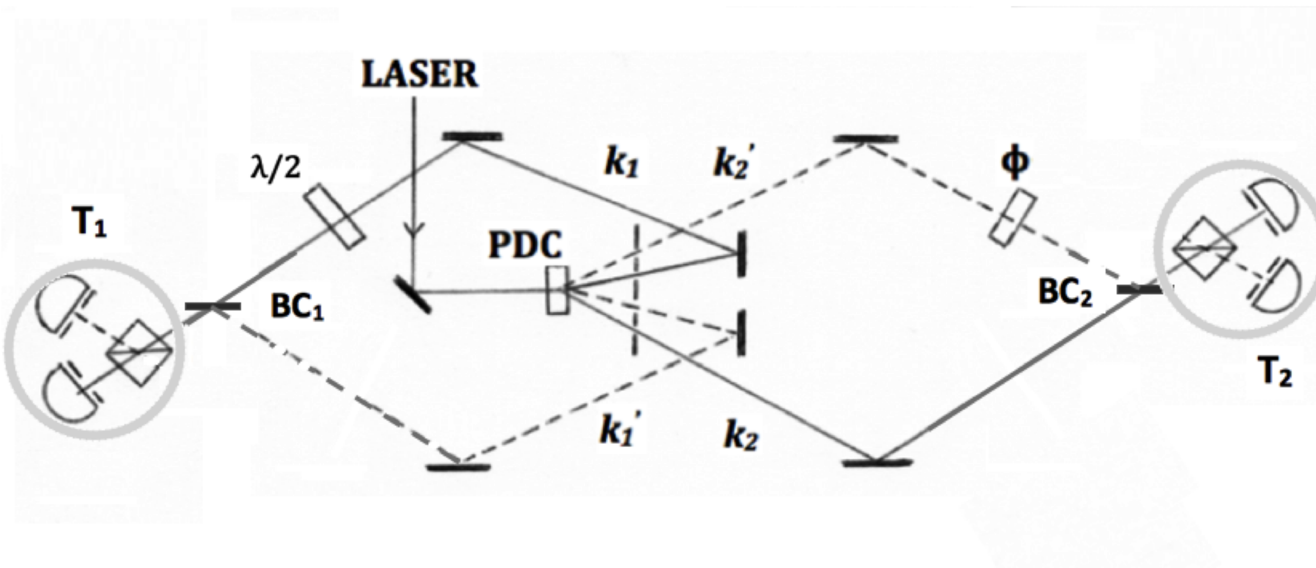

Figure 2.1: Here, a generalized experimental blueprint for a quantum eraser using hyperentanglement is given. The form taken by the beam combiners, denoted in the diagram as BC_1 and BC_2, is most simply considered as a lens, although other methods of beam recombination are possible. The k path described in the text corresponds to k_1 and k_2 in the diagram and the l path corresponds to $k_1^{'}$ and $k_2^{'}$.

Experimental Procedure

According to quantum theory, knowledge of the momentum of the photon pair gained by measuring the polarization of the signal photon should cause the interference pattern to disappear, despite the lack of explicit measurement of their starting momentum. This experiment will attempt to verify this prediction by using polarization measurement with coincidence counting of signal and idler photons reaching their detectors within a defined time window.

To begin, both HWP_k and HWP_l will be set to $\theta = 0$. This setting leaves the polarization of the photon unchanged, as shown in the calculations section. With $\theta_k = \theta_l = 0$, we predict an interference pattern will be registered between the signal and idler detectors by slightly altering the path lengths. The mirrors from which the photons are reflected can be adjusted by translational mounts, and this will give us the phase changes we need to measure an interference pattern.

After varying the phase $0 \leq \phi \leq 2\pi$ and logging the interference pattern, the next step will be to gain "which-path" information. This will be achieved by rotating HWP_k to $\theta_k = \frac{\pi}{4}$. As shown in the following section, $\theta_k = \frac{\pi}{4}$ alters the polarization of the photon by $\frac{\pi}{2}$, changing $|H\rangle$ to $|V\rangle$, and vice versa. This results in polarization measurements that would indicate which momentum the photon pair had. If the signal and idler photon have opposite polarization ($|HV\rangle$ or $|VH\rangle$), then we know that the photon pair traveled path k $\left(\theta_k = \frac{\pi}{4}\right)$. If instead they are measured to have the same polarization ($|HH\rangle$ or $|VV\rangle$) we know the pair travelled path l $(\theta_l = 0)$. This "which-path" information will cause the interference pattern to disappear. Thus, we predict we will find results that do not vary based on the phase ϕ of the mirror array.

Finally, we need to erase the "which-path" information obtained in the second phase of the experiment. In this experimental design, that will be obtained by changing HWP_l to $\theta_k = \frac{\pi}{4}$. As shown in the following sections, we no longer gain reliable "which-path" information from the polarization. This means that we will again see an interference pattern when changing ϕ in the mirror array when we observe orthogonal polarization from the PBS.

The following figures 2.2, 2.3, and 2.4 consist of various vantage points of an optics table setup of the new quantum eraser experiment using hyperentanglement. These photos were taken in the quantum information lab at Greenville University.

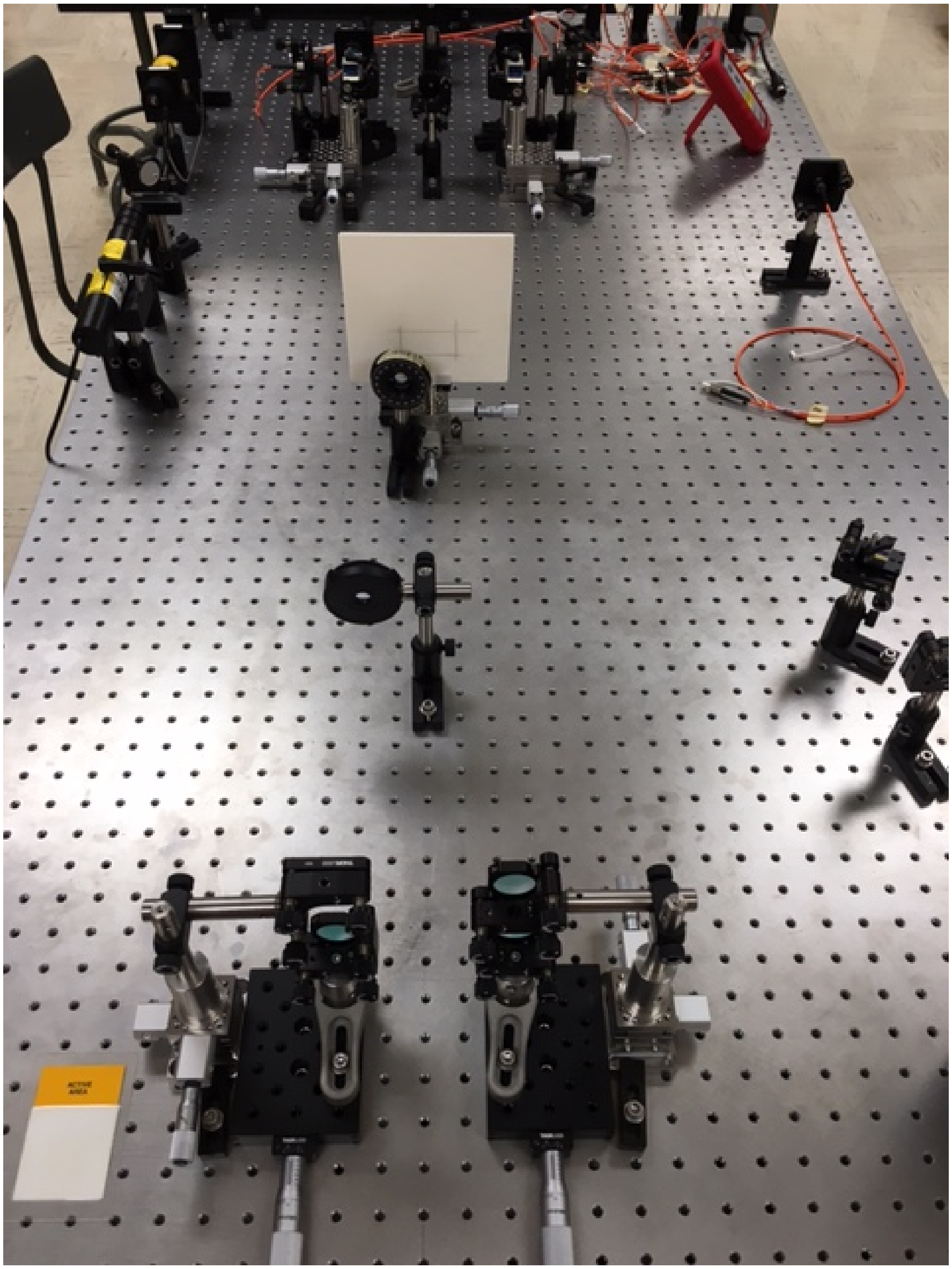

Figure 2.2: A view of an example experimental setup from behind the laser source is shown. The phase of the signal and idler photons can be adjusted via fine changes to the position of the mirrors. The half wave plates, shown between the mirror array and screen, are placed in k_1 and $k_1^{'}$ paths to adjust the phases of the signal and idler photons.

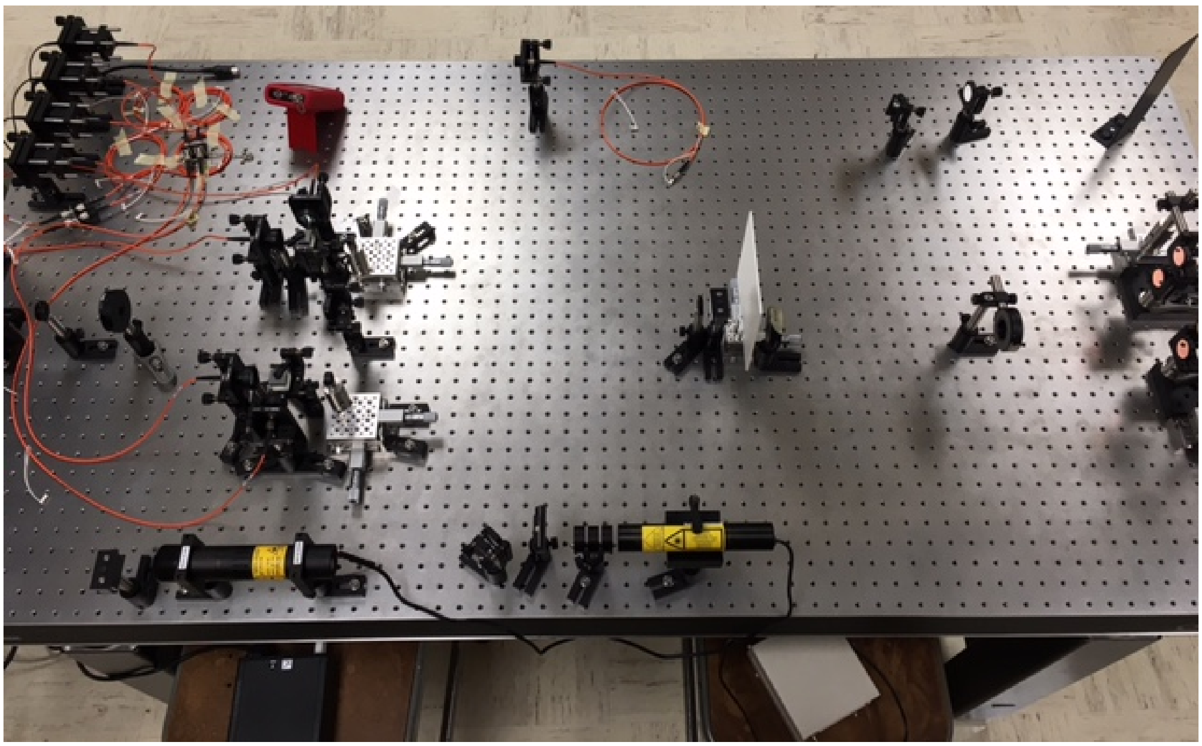

Figure 2.3: A view of an example experimental setup from behind the laser source is shown. The BBO crystal lies between the two lense-PBS-detector mounts. The photons that undergo SPDC are directed through pinholes in the white screen to the mirror array on the far right. The remaining photons from the beam are sent to a beam dump directly to the left of the screen in the image.

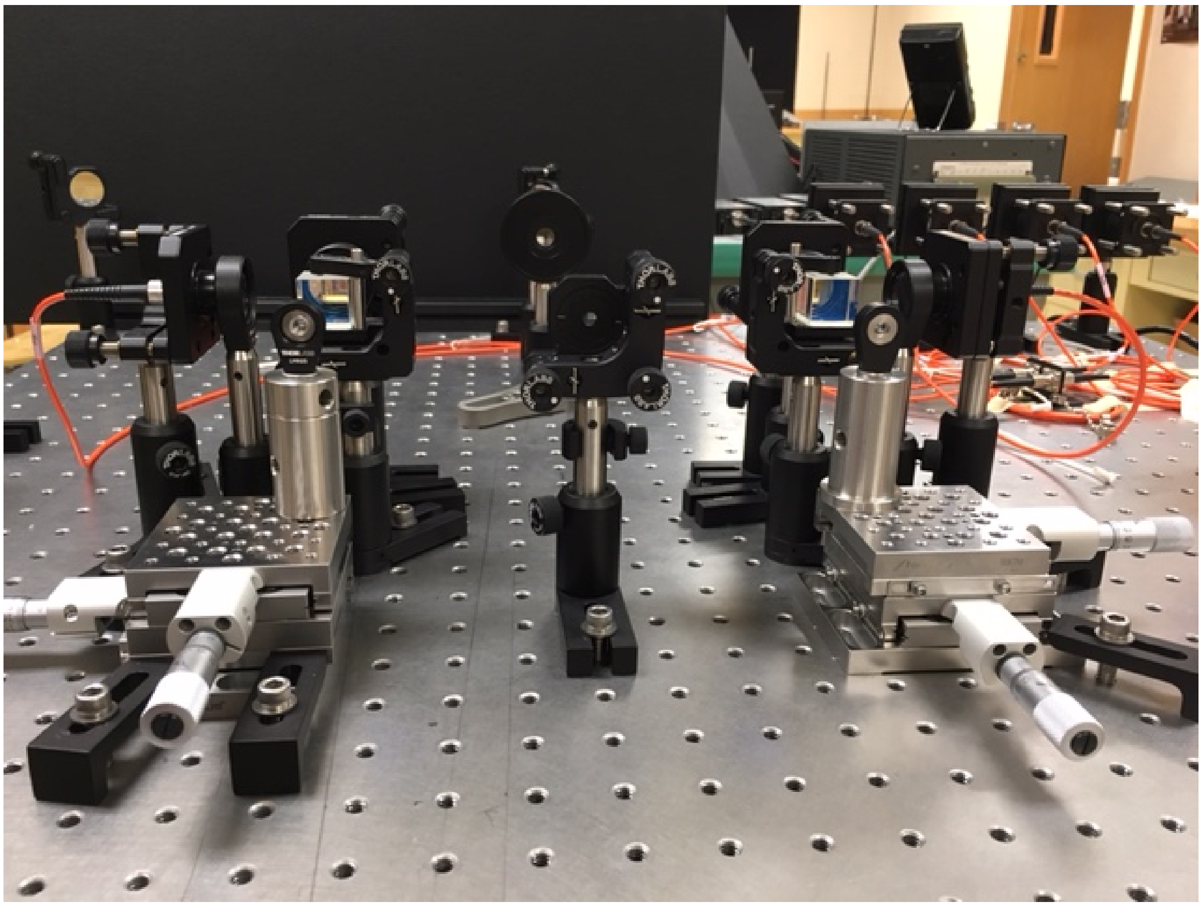

Figure 2.4: A close up view of the experimental setup's lenses, polarizing beam splitters, and detectors is shown. The lenses collimate the beams, sending them along parallel trajectories into the polarizing beam splitters. Photons from these beams are then detected, based on their polarization, by detectors 1, 2, 3, and 4.

Chapter 3

Calculations

Hyperentanglement

First, we must obtain the wave function for the hyperentangled photons. The wave function in terms of polarization is:

$$|\psi_p\rangle = \frac{1}{\sqrt{2}}\left(|h_1 h_2\rangle + |v_1 v_2\rangle\right) \tag{3.1}$$

In terms of momentum, the wave equation is:

$$|\psi_m\rangle = \frac{1}{\sqrt{2}}\left(|k_1 k_2\rangle + |l_1 l_2\rangle\right) \tag{3.2}$$

Thus, the total wave equation for the hyperentangeled photon pair is:

$$\begin{aligned} |\psi_{total}\rangle &= |\psi_p\rangle \otimes |\psi_m\rangle \\ &= \frac{1}{2}\{(|h_1 h_2\rangle + |v_1 v_2\rangle) \otimes (|k_1 k_2\rangle + |l_1 l_2\rangle)\} \end{aligned} \tag{3.3}$$

After the photon pair reflects off the mirror array, phase shifts are introduced to the wave equation.

$$
\begin{aligned}
|\psi_t\rangle &= |\psi_p\rangle \otimes |\psi_m\rangle \\
&= \frac{1}{2}\left(e^{i(\phi_1+\phi_2)}\,|h_1h_2\rangle + e^{i(\phi_3+\phi_4)}\,|v_1v_2\rangle\right) \otimes \left(|k_1k_2\rangle + |l_1l_2\rangle\right) \\
&= \frac{1}{2}(e^{i(\phi_1+\phi_2)}\,|k_1h_1, k_2h_2\rangle + e^{(\phi_1+\phi_2)}\,|k_1v_1, k_2v_2\rangle + e^{i(\phi_3+\phi_4)}\,|l_1h_1, l_2h_2\rangle \\
&\qquad +e^{i(\phi_3+\phi_4)}\,|l_1v_1, l_2v_2\rangle) \\
&= \frac{1}{2}e^{i(\phi_1+\phi_2)}[|k_1h_1, k_2h_2\rangle + |k_1v_1, k_2v_2\rangle \\
&\qquad +e^{i(\phi_3+\phi_4-\phi_1-\phi_2)}\,(|l_1h_1, l_2h_2\rangle + |l_1v_1, l_2v_2\rangle)] \\
&= \frac{1}{2}e^{i(\phi_1+\phi_2)}\left[|k_1h_1, k_2h_2\rangle + |k_1v_1, k_2v_2\rangle + e^{i\phi}\,(|l_1h_1, l_2h_2\rangle + |l_1v_1, l_2v_2\rangle)\right]
\end{aligned} \tag{3.4}
$$

Where $\phi = \phi_3 + \phi_4 - \phi_1 - \phi_2$. The coefficient $e^{i(\phi_1+\phi_2)}$ can be considered as an overall phase change, resulting in Eqaution (3.4) becoming

$$|\psi_\phi\rangle = \frac{1}{2}\{|k_1h_1, k_2h_2\rangle + |k_1v_1, k_2v_2\rangle + e^{i\phi}\,(|l_1h_1, l_2h_2\rangle + |l_1v_1, l_2v_2\rangle)\}. \tag{3.5}$$

Beam Recombination

In order to preserve the momentum entanglement, it is necessary to make the two paths indistinguishable to the detector array. This is achieved by recombining the beams.

To determine how the recombination works, we first consider a simpler model: one beam splitting evenly and recombining. Let $|\lambda\rangle$ represent the original and final wave function, and let $|k\rangle$ and $|l\rangle$ represent the wave functions of the split beams. Thus we have

$$|\lambda\rangle = \frac{1}{\sqrt{2}}(|k\rangle + |l\rangle). \tag{3.6}$$

Because $|k\rangle$ and $|l\rangle$ recombine as $|\lambda\rangle$, this implies that each of the split wave functions is some portion of the original wave function. And, since the original momentum is evenly split between $|k\rangle$ and $|l\rangle$, we see that they are each multiplied by the same coefficient. That is, $|k\rangle = |l\rangle = c\,|\lambda\rangle$ for some constant c. Thus,

$$\begin{aligned}
|\lambda\rangle &= \frac{1}{\sqrt{2}}(|k\rangle + |l\rangle) \\
|\lambda\rangle &= \frac{1}{\sqrt{2}}(c\,|\lambda\rangle + c\,|\lambda\rangle) \\
|\lambda\rangle &= \frac{2}{\sqrt{2}}c\,|\lambda\rangle \\
\implies c &= \frac{1}{\sqrt{2}} \qquad (3.7)\\
\implies |k\rangle &= |l\rangle = \frac{1}{\sqrt{2}}\,|\lambda\rangle\,. \qquad (3.8)
\end{aligned}$$

We now wish to test this coefficient in our momentum equation (ignoring polarization for the moment). As shown in Equation (3.2), we have

$$|\psi_m\rangle = \frac{1}{\sqrt{2}}\left(|k_1k_2\rangle + |l_1l_2\rangle\right).$$

In our experiment, $|k_1\rangle$ and $|l_1\rangle$ will recombine to form a new beam $|\lambda_1\rangle$. Similarly, $|k_2\rangle$ and $|l_2\rangle$ form $|\lambda_2\rangle$. Thus we have

$$\begin{aligned}
|\psi_m\rangle &= \frac{1}{\sqrt{2}}\left(|k_1k_2\rangle + |l_1l_2\rangle\right) \\
|\psi_m\rangle &= \frac{1}{\sqrt{2}}\left(c\,|\lambda_1\lambda_2\rangle + c\,|\lambda_1\lambda_2\rangle\right)
\end{aligned}$$

Using $c = \frac{1}{\sqrt{2}}$ from the first model, we get

$$\begin{aligned}
|\psi_m\rangle &= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\,|\lambda_1\lambda_2\rangle + \frac{1}{\sqrt{2}}\,|\lambda_1\lambda_2\rangle\right) \\
|\psi_m\rangle &= \frac{1}{\sqrt{2}}(\sqrt{2}\,|\lambda_1\lambda_2\rangle) \\
\implies |\psi_m\rangle &= |\lambda_1\lambda_2\rangle\,.
\end{aligned}$$

This confirms that the coefficient $c = \frac{1}{\sqrt{2}}$ found in the simple model fits our experimental design.

Now we can consider our experiment including polarization. The wave equation in Equation (3.5) after beam combination is

$$\begin{aligned} |\psi_\phi\rangle &= \frac{1}{2\sqrt{2}}\{|h_1h_2\rangle \otimes |\lambda_1\lambda_2\rangle + e^{i\phi}|h_1h_2\rangle \otimes |\lambda_1\lambda_2\rangle + |v_1v_2\rangle \otimes |\lambda_1\lambda_2\rangle + e^{i\phi}|v_1v_2\rangle \otimes |\lambda_1\lambda_2\rangle\} \\ &= \frac{1}{2\sqrt{2}}\{\left(1+e^{i\phi}\right)|h_1h_2\rangle + \left(1+e^{i\phi}\right)|v_1v_2\rangle\} \\ &= \frac{1}{2\sqrt{2}}\left(1+e^{i\phi}\right)\{|h_1h_2\rangle + |v_1v_2\rangle\} = |\psi'_\phi\rangle\,. \end{aligned} \tag{3.9}$$

Expectation Value: Stage 1

The general operator for a two-channel detector is given by

$$\hat{T} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \tag{3.10}$$

Setting $\theta = 0$, we obtain the two-channel operator

$$\hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.11}$$

The expectation value with the two-channel operators $\langle \hat{T}_1 \otimes \hat{T}_2 \rangle$ is thus:

$$
\begin{aligned}
\langle \hat{T}_1 \otimes \hat{T}_2 \rangle &= \langle \psi'_\phi | \, \hat{T}_1 \otimes \hat{T}_2 \, | \psi'_\phi \rangle \\
&= \left[\frac{1}{2\sqrt{2}} \left(1 + e^{-i\phi}\right) \{ \langle h_1 h_2 | + \langle v_1 v_2 | \} \right] \hat{T}_1 \otimes \hat{T}_2 \left[\frac{1}{2\sqrt{2}} \left(1 + e^{i\phi}\right) \{ | h_1 h_2 \rangle + | v_1 v_2 \rangle \} \right] \\
&= \frac{1}{8} \left(1 + e^{-i\phi}\right) \left(1 + e^{i\phi}\right) \{ \langle h_1 h_2 | + \langle v_1 v_2 | \} \hat{T}_1 \otimes \hat{T}_2 \{ | h_1 h_2 \rangle + | v_1 v_2 \rangle \} \\
&= \frac{1}{8} \left(2 + e^{-i\phi} + e^{i\phi}\right) \{ \langle h_1 h_2 | + \langle v_1 v_2 | \} \{ | h_1 h_2 \rangle + | v_1 v_2 \rangle \} \\
&= \frac{1}{8} \left(2 + e^{-i\phi} + e^{i\phi}\right) (2) \\
&= \frac{1}{4} \left[2 + 2 \left(\frac{e^{i\phi} + e^{-i\phi}}{2} \right) \right] \\
&= \frac{1}{2} \left(1 + \cos\phi\right) \\
&= \cos^2 \left(\frac{\phi}{2} \right) \qquad (3.12) \\
&= [0, 1]
\end{aligned}
$$

This gives us an expectation value that ranges from 0 to 1 dependent on ϕ, indicating an interference pattern in the experimental results.

Half-Wave Plate

In the second stage of our experiment, we add a half-wave plate in the path of one of the photons. The matrix operator for the half-wave plate is

$$\hat{J}_{\frac{\lambda}{2}} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}.$$

With $\theta = \frac{\pi}{4}$ the matrix becomes

$$\hat{J}_{\frac{\lambda}{2}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \hat{Z}. \qquad (3.13)$$

The effect of the half-wave plate on horizontal and vertical polarization is

$$\hat{Z}\,|h\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |v\rangle$$
$$\hat{Z}\,|v\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |h\rangle\,.$$

If we place the half-wave plate on the k_2 path after the mirror array, the equation $|\psi_\phi\rangle$ becomes:

$$\hat{Z}_k\,|\psi_\phi\rangle = \frac{1}{2}\{|k_1h_1, k_2v_2\rangle + |k_1v_1, k_2h_2\rangle + e^{i\phi}(|l_1h_1, l_2h_2\rangle + |l_1v_1, l_2v_2\rangle)\}. \tag{3.14}$$

We name Equation (3.14) as $|\psi_{\frac{\lambda}{2}}\rangle$. Following the same beam combination as in the derivation of Equation (3.8) we find:

$$|\psi'_{\frac{\lambda}{2}}\rangle = \frac{1}{2\sqrt{2}}\{|h_1v_2\rangle + |v_1h_2\rangle + e^{i\phi}(|h_1h_2\rangle + |v_1v_2\rangle)\}. \tag{3.15}$$

Expectation Value: Stage 2

We now wish to find the expectation value of the two-channel detector array $\langle\hat{T}_1 \otimes \hat{T}_2\rangle$.

$$
\begin{aligned}
\langle \hat{T}_1 \otimes \hat{T}_2 \rangle &= \langle \psi'_{\frac{\lambda}{2}} |\, \hat{T}_1 \otimes \hat{T}_2 \,| \psi'_{\frac{\lambda}{2}} \rangle \\
&= \frac{1}{8}\{\langle h_1 v_2| + \langle v_1 h_2| + e^{-i\phi}(\langle h_1 h_2| + \langle v_1 v_2|)\}\hat{T}_1 \otimes \hat{T}_2\{|h_1 v_2\rangle + |v_1 h_2\rangle \\
&\qquad\quad + e^{i\phi}(|h_1 h_2\rangle + |v_1 v_2\rangle)\} \\
&= \frac{1}{8}\{\langle h_1 v_2| + \langle v_1 h_2| + e^{-i\phi}(\langle h_1 h_2| + \langle v_1 v_2|)\}\{-|h_1 v_2\rangle - |v_1 h_2\rangle \\
&\qquad\quad + e^{i\phi}(|h_1 h_2\rangle + |v_1 v_2\rangle)\} \\
&= \frac{1}{8}(-1 - 1 + 1 + 1) \\
&= 0
\end{aligned}
$$

In this stage of the experiment, the expectation value is independent of the phase change due to the mirror array, eliminating the interference pattern. This is consistent with other quantum eraser experiments after “which-path” information is gained.

Expectation Value: Stage 3

Finally, we wish to reintroduce the interference pattern seen in the first stage of the experiment. To do this we adjust HWP$_k$ to $\theta_k = \frac{\pi}{4}$. This results in the wave equation of the final stage:

$$
|\psi_f\rangle = \frac{1}{2}\{|k_1 h_1, k_2 v_2\rangle + |k_1 v_1, k_2 h_2\rangle + e^{i\phi}(|l_1 h_1, l_2 v_2\rangle + |l_1 v_1, l_2 h_2\rangle)\}. \tag{3.16}
$$

After beam recombination we arrive at

$$
\begin{aligned}
|\psi_f\rangle &= \frac{1}{2\sqrt{2}}\{|h_1 v_2\rangle + |v_1 h_2\rangle + e^{i\phi}(|h_1 v_2\rangle + |v_1 h_2\rangle)\} \\
&= \frac{1}{2\sqrt{2}}\{\left(1 + e^{i\phi}\right)|h_1 v_2\rangle + \left(1 + e^{i\phi}\right)|v_1 h_2\rangle\} \\
&= \frac{1}{2\sqrt{2}}\left(1 + e^{i\phi}\right)\{|h_1 v_2\rangle + |v_1 h_2\rangle\} = |\psi'_f\rangle\,.
\end{aligned} \tag{3.17}
$$

As in the previous stages, we wish to compute $\langle \hat{T}_1 \otimes \hat{T}_2 \rangle$.

$$
\begin{aligned}
\langle \hat{T}_1 \otimes \hat{T}_2 \rangle &= \langle \psi'_f | \, \hat{T}_1 \otimes \hat{T}_2 \, | \psi'_f \rangle \\
&= \frac{1}{8} \left(1 + e^{-i\phi}\right) \{\langle h_1 v_2| + \langle v_1 h_2|\} \hat{T}_1 \otimes \hat{T}_2 \left(1 + e^{i\phi}\right) \{|h_1 v_2\rangle + |v_1 h_2\rangle\} \\
&= \frac{1}{8} \left(1 + e^{-i\phi}\right) \left(1 + e^{i\phi}\right) \{\langle h_1 v_2| + \langle v_1 h_2|\} \hat{T}_1 \otimes \hat{T}_2 \{|h_1 v_2\rangle + |v_1 h_2\rangle\} \\
&= \frac{1}{8} \left(2 + e^{-i\phi} + e^{i\phi}\right) \{\langle h_1 v_2| + \langle v_1 h_2|\} \{- |h_1 v_2\rangle - |v_1 h_2\rangle\} \\
&= \frac{1}{8} \left(2 + e^{-i\phi} + e^{i\phi}\right) (-2) \\
&= -\frac{1}{4} \left[2 + 2 \left(\frac{e^{i\phi} + e^{-i\phi}}{2}\right)\right] \\
&= -\frac{1}{2} \left(1 + \cos\phi\right) \\
&= -\cos^2 \left(\frac{\phi}{2}\right) \\
&= [-1, 0]
\end{aligned}
$$

This confirms that an interference pattern will be reintroduced as in the first stage of the experiment, but rather than the “fringes” seen in Stage 1, here we have “antifringes”.

Chapter 4

Conclusions

As described in section 1.3, a quantum eraser typically uses a measurement of one of two entangled photons to determine the quantum state of the other. For example, if Type I SPDC was used to create two entangeled photons, and one was measured to have vertical polarization, then the other must also have vertical polarizaiton. However, with this experimental design, it is possible to determine not only the polarization of the unmeasured photon based on measurement of its entangled twin, but also to determine "which-path" information about the entangled momentum states. This means that hyperentanglement can allow us to know the momentum and polarization of a photon by measuring the polarization of its entangled twin.

By using measurement of polarization to determine the momentum of the photons upon leaving the BBO crystal, we can show that in hyperentangled states, knowledge of one quantum state of a photon can lead to knowledge of another; in this case, knowledge of polarization leads to knowledge of momentum. It is thus significant because it shows that the complete quantum state of one particle can be determined via measurement of a single quantum state of another particle.

References

[1] Einstein, A., Podolsky, B., and Rosen, N., 1935, Phys. Rev., 47, 777-80.

[2] Neves, L., et al, 2009, New J. Phys., 11, 073035.

[3] Ashby, James M. and Schwarz, Peter D. and Schosshauer, Maximilian, "Delayed-choice quantum eraser for the undergraduate laboratory," Am. J. Phys. 84(2), 95-105(2016), https://doi.org/10.1119/1.4938151.

[4] Rueckner, Wolfgang and Peidle, Joseph, "Young's double-slit experiment with single photons and quantum eraser," Am. J. Phys. 81(12), 951-958(2013), https://doi.org/10.1119/1.4819882.

[5] Schneider, Mark B. and LaPuma, Indhira A., "A simple experiment for discussion of quantum interference and which-way measurement," Am. J. Phys. 70(3), 266-271(2002), https://doi.org/10.1119/1.1450558.

[6] Jordan, Thomas F., "Choosing and rechoosing to have or have not interference," Am. J. Phys. 69(2), 155-157(2001), https://doi.org/10.1119/1.1290253.

[7] Englert, Berthold-Georg and Scully, Marlan O. and Walther, Herbert, "Quantum erasure in double-slit interferometers with which-way detectors," Am. J. Phys. 67(4), 325-329(1999), https://doi.org/10.1119/1.19257.

[8] Holbrow,C. H. and Galvez,E. and Parks,M. E., “Photon quantum mechanics and beam splitters,” Am. J. Phys. 79(3), 260-265(2002), https://doi.org/10.1119/1.1432972.

[9] Garappa, N., ”On the Foundations of Quantum Mechanics: Wave-Particle Non-Duality and the Nature of Physical Reality”. (2017), arXiv preprint arXiv:1710.09270.